Influence of Cheese Ripening on the Viscoelastic Behaviour of Edam Cheese

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Abstract

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The paper deals with the study of the effect of cheese ripening on parameters of a rheological model of cheese mechanical behaviour. The Edam cheese has been tested by the method of the Hopkinson Split Pressure Bar. The original method of the evaluation of viscoelastic properties has been used. The rheological model of the three element linear viscoelastic body, so called "standard linear solid" has been used. This model successfully describes the experimentally observed deformation behaviour of cheese specimens. The effect of the time of cheese ripening on the parameters of the rheological model has been demonstrated.

Keywords: Edam cheese; ripening; standard model; model parameters; sensory evaluation

The rheological properties of cheeses are significantly connected with their final quality (KFOURY *et al.* 1989). Like most of solid biological materials cheeses exhibit the viscoelastic behaviour. This means that their behaviour under some mechanical loading consists of solid (elastic) and fluid (viscous) behaviour. The viscoelastic materials are those in which the relationship between stress and strain depends on time. There are many models of the viscoelastic behaviour of materials, see e.g. LAKES (1999) for a review. The choice of a viscoelastic model and the evaluation of its parameters is based on the use of some phenomena which occur in viscoelastic materials. The main phenomena are:

– Creep – if the stress is held constant, the strain increases with time;
– Relaxation – the stress decreases with time at constant strain;
– The effective stiffness depends on the rate of load application;
– If cyclic loading is applied, hysteresis occurs;
– Acoustic waves experience damping;
– Rebound of an object following an impact is less than 100%;

and many others (LAKES 1999).

The development of methods of the evaluation of cheese viscoelastic properties is described e.g. by KONSTANCE and HOLSINGER (1992). Recently a new method of the evaluation of these properties has been developed (BUCHAR 1996). The use of this method for the evaluation of some cheeses outlined its potential abilities. In the given paper we have focused on the study of cheese ripening, on the viscoelastic behaviour of Edam cheese. The obtained viscoelastic properties have been compared with sensory characteristics. Owing to some exclusivity of the used procedure the relatively detailed description of its theoretical base is described.

Theoretical Background

In the previous paper (BUCHAR 1996) it was found that the behaviour of the tested cheeses was described by the model of the standard linear body (HAMAN & ZDANOWICZ 1996). A schematic of this model is shown in Fig. 1. This model contains three elements. The springs are perfectly elastic ($\sigma = E\,\varepsilon$). The dashpot is perfectly viscous ($\sigma = \eta\, d\varepsilon/dt$), where η is the dynamic viscosity. The constitutive equation connected with this model has a form:

$$\sigma(t) = E_1\varepsilon(t) + E_2\int_0^t \left(\frac{d\varepsilon}{dt}\right).\exp\left(-\frac{t-\tau}{\theta}\right)d\tau \qquad [1]$$

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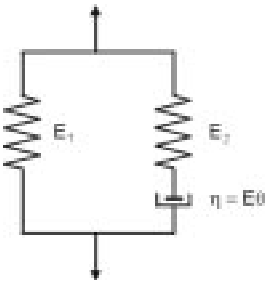

Fig. 1. Schematic of the standard linear solid

or in a differential form:

$$\frac{d\sigma(t)}{dt} + \frac{\sigma(t)}{\theta} = (E_1 + E_2)\frac{d\varepsilon(t)}{dt} + \frac{E_1\varepsilon}{\theta} \qquad [2]$$

where: q – relaxation time

$$\theta = \eta/E_2$$

The main problem consists in the evaluation of the parameters E_1, E_2 and θ. The procedure suggested in BUCHAR (1996) uses the stress wave propagation. If we take the one-dimensional stress state, the wave has a form:

$$u(x,t) = A.\exp(-\alpha.x).\exp[-I(\omega t - kx)]$$
$$k(\omega) = \frac{\omega}{c(\omega)} \qquad [3]$$

where: u – particle displacement
x – distance in the direction of the wave propagation
ω – angular frequency
I – denotes the imaginary unit
$\alpha = \alpha(\omega)$ – damping coefficient
c(ω) – phase velocity
$k(\omega)$ – wave number

The dependence of $k(\omega)$ or phase velocity $c(\omega)$ on the frequency describes the dispersion of waves, and the dependency $\alpha = \alpha(\omega)$ describes the attenuation and dissipation of waves. If we use the main equations describing the one-dimensional wave motion (see e.g. ACHENBACH 1973) we can express Eq. [2] in terms of particle displacement as:

$$\rho\frac{\partial^3 u(x,t)}{\partial t^3} + \rho\frac{\frac{\partial^2 u(x,t)}{\partial t^2}}{\theta} = E_2\frac{\partial^3 u(x,t)}{\partial x^2 \partial t} + \frac{E_1\frac{\partial^2 u(x,t)}{\partial x^2}}{\theta} \qquad [4]$$

where: ρ – material density

If we insert Eq. [3] into Eq. [4], we obtain a complex equation. Its solution for the real and imaginary part (BUCHAR 1996) leads to the following expressions for the damping coefficient and wave number:

$$\alpha(\omega)^2 = \frac{\rho\omega^2}{2E_1}\left\{\sqrt{\frac{1+\omega^2\theta^2}{1+\left(1+\frac{E_2}{E_1}\right)^2\omega^2\theta^2}} - \frac{1+\left(1+\frac{E_2}{E_1}\right)\omega^2\theta^2}{1+\left(1+\frac{E_2}{E_1}\right)^2\omega^2\theta^2}\right\}$$

$$k(\omega)^2 = \frac{\rho\omega^2}{2E_1}\left\{\sqrt{\frac{1+\omega^2\theta^2}{1+\left(1+\frac{E_2}{E_1}\right)^2\omega^2\theta^2}} + \frac{1+\left(1+\frac{E_2}{E_1}\right)\omega^2\theta^2}{1+\left(1+\frac{E_2}{E_1}\right)^2\omega^2\theta^2}\right\} \qquad [5]$$

Let us follow the dispersion and attenuation of the high frequency wave, i.e. $\omega^2 >> 1/\theta^2$. Equation [5] shows that k approaches

$$k^2 \approx \frac{\rho\omega^2}{E_1 + E_2} \qquad [6]$$

Equation [6] expresses that the phase velocity of the high frequency wave is independent of the frequency and it is equal to the velocity of an elastic wave:

$$c_\infty^2 = \frac{\omega^2}{k^2} = \frac{E_1 + E_2}{\rho} \qquad [7]$$

The damping coefficient a of high frequency waves has the following form:

$$\alpha^2 = \frac{\rho E_2^2}{4\theta^2(E_1 + E_2)^3} \qquad [8]$$

That means that the attenuation of high frequency waves does not depend on the frequency. If we use Eq. [7] and the definition of θ, we obtain the expression:

$$\alpha = \frac{E_2}{2\theta c_\infty(E_1 + E_2)} = \frac{\rho c_\infty}{2\eta\left(1 + \frac{E_2}{E_1}\right)^2} \qquad [9]$$

It is obvious that the evaluation of the viscoelastic model parameters, E_1, E_2, η needs an experimental method which enables to determine the frequency dependence of the attenuation and phase velocity of the wave. There are many experimental procedures which use the wave propagation. One of them is the split Hopkinson bar technique (SHBT). This method has been widely used by numerous investigators in studies of the dynamic mechanical behaviour of materials since it was proposed by KOLSKY (1949). Basically, this technique consists of sandwiching a short cylindrical specimen of test material between two long elastic bars as shown in Fig. 2.

A stress pulse $\sigma_I(t)$ is initiated in the first bar by the impact of a striker bar. This incident stress pulse loads the test specimen, and as a result of its interaction with the specimen, a reflected stress pulse $\sigma_R(t)$ and a transmitted stress pulse $\sigma_T(t)$ are generated at the left and at

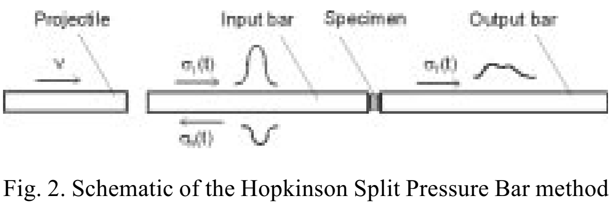

Fig. 2. Schematic of the Hopkinson Split Pressure Bar method

the right interfaces of the specimen, respectively. If we use the obvious assumptions (BUCHAR & BILEK 1984) we may obtain some material characteristics, e.g.:
– stress in the specimen $\sigma(t) = \sigma_T(t) = \sigma_I(t) + \sigma_R(t)$
– strain rate and strain in the specimen:

$$\acute{\imath}(t) = \frac{\acute{o}_I(t) - \sigma_R(t) - \acute{o}_T(t)}{Z_b l_o}$$

$$\acute{\imath}(t) = \int \acute{\imath}\, \mathrm{d}t$$

where: Z_b – acoustic impedance of the bar
$Z_b = \rho_b c_b$, ρ_b – material density
c_b – one-dimensional wave velocity
l_o – specimen length

For our purposes it is convenient to use the frequency domain where the stress pulses are expressed as:

$$\sigma_I(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} S_I(\omega).\exp(I\omega t)d\omega$$

$$\sigma_T(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} S_T(\omega).\exp(I\omega t)d\omega$$

$$\sigma_{RI}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} S_R(\omega).\exp(I\omega t)d\omega$$

$$S_{I,T,R} = \int_{-\infty}^{+\infty} \sigma_{I,T,R} \exp(-I\omega t)dt$$

where: S – spectral function

The corresponding spectral functions enable to introduce the functions describing the transmission and reflection of the stress pulse in the frequency domain as:

$$T(\omega) = \frac{S_T(\omega)}{S_I(\omega)},\ R(\omega) = \frac{S_R(\omega)}{S_I(\omega)} \qquad [10]$$

If the specimen behaves as elastic body, the function T can be expressed as (BREKHOVSKICH 1973):

$$Z_s(\omega) = \frac{\rho\omega}{\dfrac{\omega}{c(\omega)} + I\alpha(\omega)} \qquad [11]$$

where: Z_s – acoustic impedance of the specimen

Its value is independent of the frequency. If the specimen is from a viscoelastic material, the acoustic impedance Z_s is a complex number dependent on the frequency (AKI & RICHARDSON 1980).

$$T(\omega) = \frac{4Z_b Z_s \exp(I\omega l_o)}{(Z_b + Z_s)^2\left[1 - \left(\dfrac{Z_s - Z_b}{Z_s + Z_b}\right)^2 \exp(2I\omega l_o)\right]} \qquad [12]$$

The transmission of the stress pulse is then given by the function:

$$T(\varpi) = \frac{4Z_s Z_b \exp\,(I\Theta l_o)}{(Z_s + Z_b)^2\left\{1 - \left(\dfrac{Z_s - Z_b}{Z_s + Z_b}\right)^2 \exp\,(I2\Theta l_o)\right\}} \qquad [13]$$

$$\Theta = \frac{\omega}{c(\omega)} + I\alpha(\omega)$$

These relations enable to obtain the frequency dependence of the attenuation and phase velocity. If we perform the experiment shown in Fig. 2, we obtain the functions $\sigma_I(t)$ and $\sigma_T(t)$. Then we perform the Fourier transform in order to obtain the spectral functions S_I and S_T and the function of T – see Eq. [10]. If we compare the experimental values of T with the theoretical values given by Eq. [13] we can obtain Z_s and Θ. In this way we can obtain the values of $c(\omega)$ and $\alpha(\omega)$ at different frequencies. Using Eqs. [5] and [7] we may obtain the values of parameters E_1, E_2 and η.

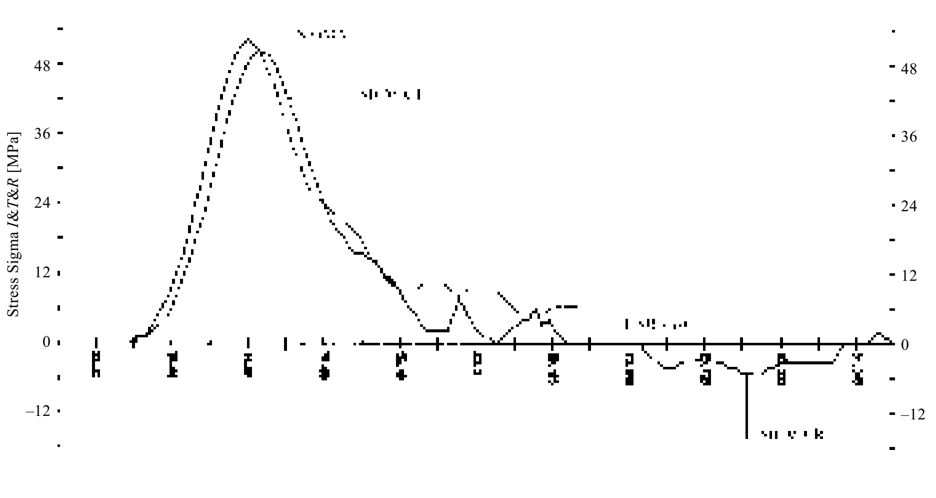

Fig. 3. Example of the experimental record of the stress pulses $\sigma_I(t)$, $\sigma_T(t)$ and $\sigma_R(t)$. Impact velocity of the striker bar was 24.87 m/s (one week of ripening)

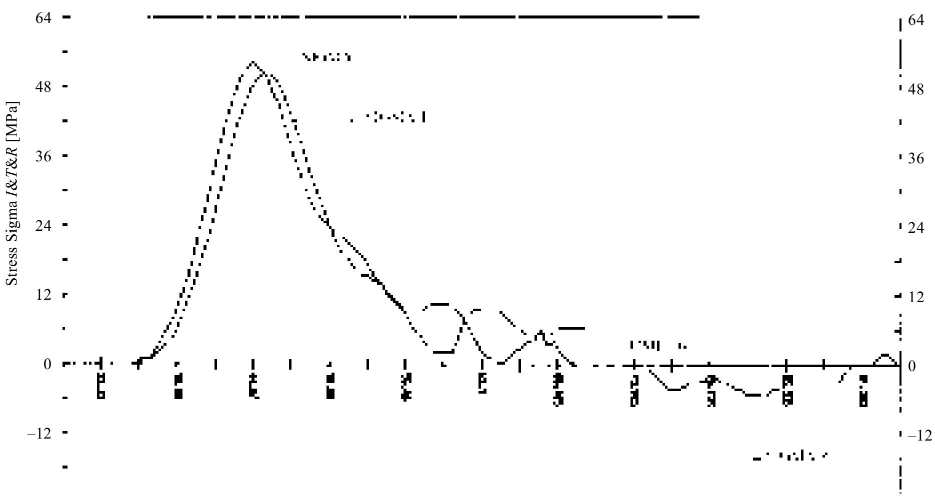

Fig. 4. The strain and strain rate as functions of time. These time dependences have been computed for the stress pulses displayed in Fig. 3

MATERIAL AND PROCEDURE

Edam cheese of different age (1–59 weeks) was analysed. Samples were prepared from commercially manufactured Edam bricks; with 30% and 45% fat in dry matter (FDM). A dairy plant processes about 50 000 l of milk a day in a standard manufacturing schedule. Approximately ten sets of 210 Edam bricks (in each batch) are produced. Immediately after salting, 6 bricks (randomly chosen) from different batches were cut to 5 slices (each about 0.5 kg) and vacuum-packaged in cryovac foil. Samples were stored at 10°C for measurements after given periods. From the block of cheese the specimens of 14 mm in diameter and 7 mm in thickness were prepared. These specimens were tested using the method shown in Fig. 2. The test bars are made of PMMA. The acoustic impedance of PMMA is 3.18 MPas/m. The loading stress pulses were initiated by the impact of a short PMMA bar (20 mm in length). An example of the experimental record of the stress pulses $\sigma_I(t)$, $\sigma_T(t)$ and $\sigma_R(t)$ is given in Fig. 3.

The corresponding strain and strain rate are shown in Fig. 4.

These time dependences were computed for the stress pulses displayed in Fig. 3.

These pulses, namely $\sigma_I(t)$ and $\sigma_T(t)$ were expressed in form of Fourier integral. In this way the function $T(\omega)$ which characterises the transmission of the stress pulse through the specimen was obtained. This function is complex, i.e.

$$T(\omega = T_1(\omega) + I * T_2(\omega) = T \exp(I * \varphi)$$
$$T = \sqrt{T_1^2 + T_2^2} \qquad \varphi = arctg(\frac{T_2}{T_2}) \qquad [14]$$

An example of the frequency dependence of the amplitude of the transmission function T is shown in Fig. 5.

Now we can use the theoretical value of T and by a comparison with the experimental values one can obtain

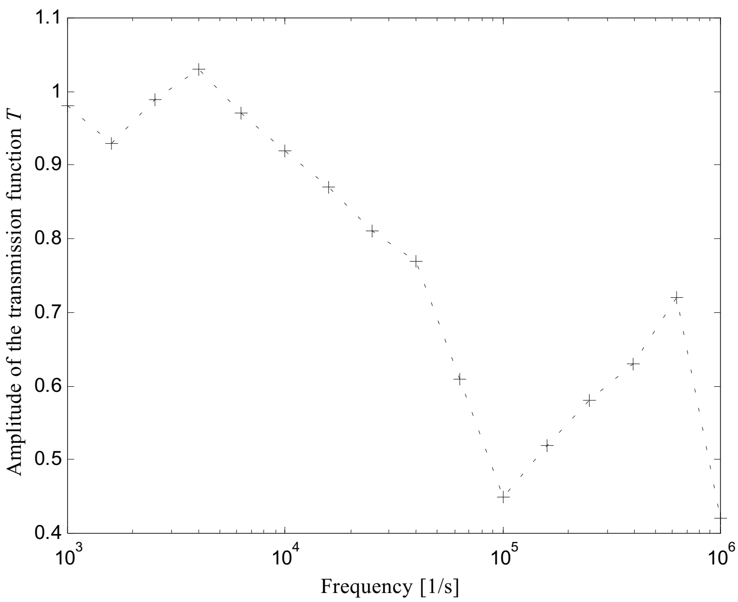

Fig. 5. Experimental values of the amplitude of the transmission function T. The data have been obtained from the stress pulses reported in Fig. 3

the value of α(ω) and c(ω). The procedure is very complicated and it is convenient to use the MAPLE software or any similar program. In this way we obtain the left sides of Equations [5]. These are two equations for three unknown parameters E_1, E_2 and θ. It means we need another equation. We can use Equation [7] which describes the value of c for the infinite frequency. These three equations, i.e. [5] and [7], enable to obtain the parameters of the rheological model which is shown in Fig. 1. These equations are highly nonlinear and the use of the software mentioned above is very convenient. This procedure was applied to the experimental results obtained for the specimens of the Edam cheese. For every stage of cheese ripening these parameters were determined for three different incident stress pulses (different velocities of the impact of the striker – see Fig. 2). The resulting value of the rheological parameter represents an average of these values.

RESULTS AND DISCUSSION

The parameters of the rheological model shown in Fig. 1 are given in Table 1.

In the given table the values of stress wave propagation at infinite frequency – see Eq. [7] – are also given together with the values of viscosity (Fig. 1). The quantities given in Table 1 decrease with the time of cheese ripening. This tendency is manifested in Figs. 6–9.

Time of the Cheese Ripening: The obtained results enable to evaluate the dependence of damping and phase velocity on the frequency. The phase velocity c(ω) and damping α(ω) increase with the frequency up to some limit value given by Eqs. [7] and [8]. The phase velocity c(ω) decreases with the time of cheese ripening (Fig. 9). The damping coefficient at infinite frequency increases with the frequency as shown in Fig. 10.

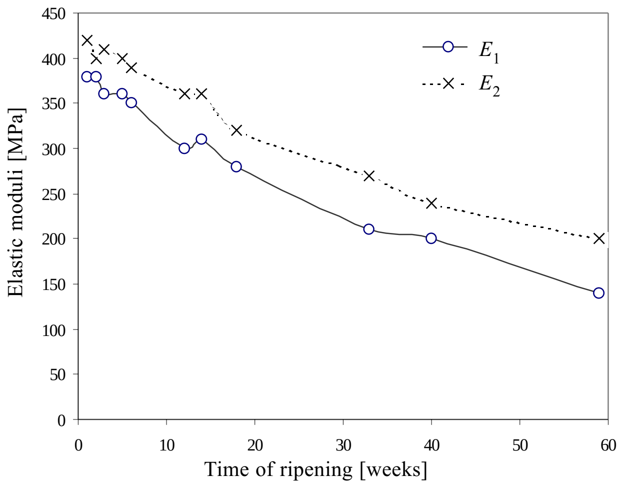

Fig. 6. The effect of the time of cheese ripening on the elastic moduli

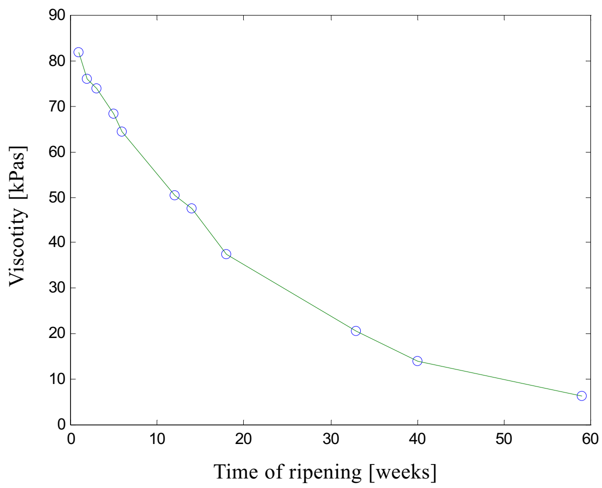

Fig. 8. The effect of the time of cheese ripening on viscosity

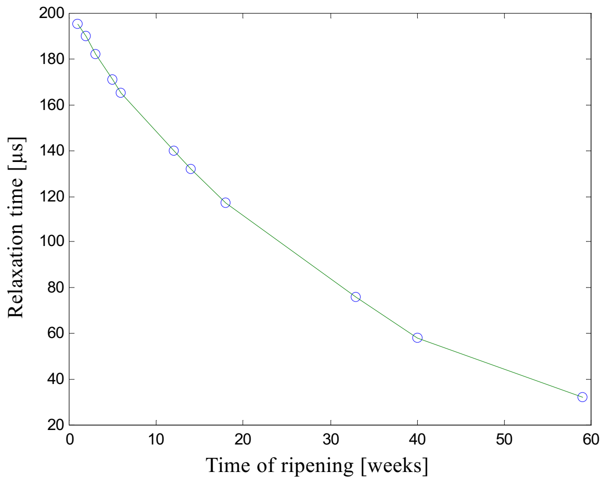

Fig. 7. The effect of the time of cheese ripening on the relaxation time

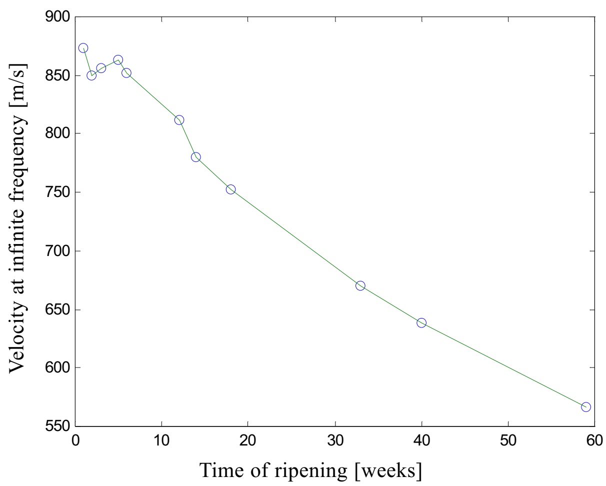

Fig. 9. The effect of the time of cheese ripening on the velocities of the high frequency waves

Table 1. The parameters of the Edam cheese rheological model

Time of ripening [weeks]	Density [kg/m^3]	E_1 [MPa]	E_2 [MPa]	θ [µs]	c_∞ [m/s]	η [kPas]
1	1070	380	420	195	873	81.9
2	1080	380	400	190	850	76.0
3	1050	360	410	182	856	73.8
5	1020	360	400	171	863	68.4
6	1020	350	390	165	852	64.4
12	1000	30	360	140	812	50.4
14	1100	310	360	132	780	47.5
18	1060	280	320	117	752	37.4
33	1070	210	270	76	670	20.5
40	1080	200	240	58	638	13.9
59	1060	140	200	32	566	6.4

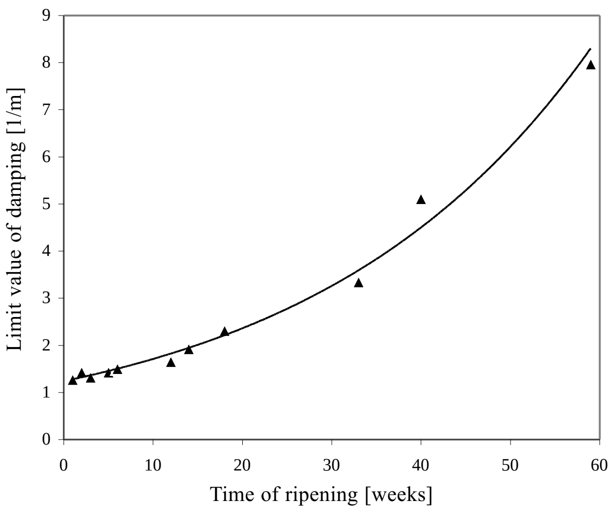

Fig. 10. The frequency dependence of the limit value of damping coefficient (see Eq. 8)

The increase is remarkable namely for a time higher than about 18 weeks. The knowledge of this coefficient is very useful for the description of stress pulse propagation in the viscoelastic body. Let us take a block of viscoelastic body which is impacted by a projectile moving with some velocity. If we use a very simple description of this process, the dependence of the stress in the block on the distance *x*, in the direction of the wave propagation is given by the equation (e.g. WANG *et al*. 1994):

$$\sigma(x) = \sigma_o \exp\left[-\frac{\rho c_\infty}{2\eta(1+\frac{E_1}{E_2})^2}x\right] = \exp(-\alpha x)$$

where: α – given by the Eq. [8]

It means that the stress pulse is attenuated during its propagation. This attenuation increases with the time of cheese ripening.

References

ACHENBACH J.D. (1973): Waves in Elastic Solids. Amsterdam, North Holl. Publ.

AKI K., RICHARDSON P.G. (1980): Quantitative seismology. San Francisco, W.H. Freemand Co.

BREKHOVSKIH L. (1973): Volny v sloistych sredach. Moskva, Nauka.

BUCHAR J., BÍLEK Z. (1984): Chování kovových materiálů při vysokých rychlostech deformace. Praha, Academia – NČSAV.

BUCHAR J. (1996): Experimental and theoretical study of the rheological properties of selected cheeses. In: BLAHOVEC J. (Ed.): Stress Relaxation in Solids of Biological Origin. Prague, Czech Univ. Agric. Press: 78–86.

HAMAN J., ZDANOWICZ A. (1968): O potrzebie roszerzania studiów nad reologia materialow w rolnictwie. Rocz. Nauk Rol., **2**: 68–82.

KFOURY M., MPAGANE M., HARDY J. (1989): Effect of cheese ripening on rheological properties Camembert and Saint-Paulin cheeses. Lait, **69**: 137–149.

KOLSKY H. (1949): An investigation of the mechanical characteristics of materials at very high strain rates. Proc. Phys. Soc., **B62**: 676–688.

KONSTANCE R.P., HOLSINGER V.H. (1993): Development of rheological test methods for cheese. Food Technol., **46**: 105–109.

LAKES R.S. (1999): Viscoelastic solids. New York, CRC Press.

WANG L., LABIBES K., AZARI Z., PLUVINAGE G. (1994): Generalization of split Hopkinson bar technique to use viscoelastic bar. Int. J. Impact Engng, **15**: 669–686.

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Souhrn

BUCHAR J., KUBIŠ I., GAJDŮŠEK S., KŘIVÁNEK I. (2001): **Vliv zrání sýra na vazkopružné vlastnosti eidamského sýra**. Czech J. Food Sci., **19**: 1–7.

Byl sledován vliv doby zrání na parametry reologického modelu eidamského sýra. Vzorky sýra byly zatěžovány metodou Hopkinsonovy měrné dělené tyče. Byla navržena původní metoda hodnocení výsledků tohoto způsobu zatěžování, která umožňuje sledovat vazkopružné vlastnosti. Ukazuje se, že tyto vlastnosti jsou popsány v rámci tzv. standardního modelu lineárního tělesa. Je vyhodnocen vliv doby zrání na parametry tohoto modelu.

Klíčová slova: Eidamský sýr; zrání; standardní model; útlum; elasticita

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